The most remarkable find in this region, however, was made on the Whitewater river, twenty miles east of Wichita, and seems to date back in geologic times prior to the last general submergence of the country. At a point near Augusta the river has gradually deflected to the east cutting its way slowly into the valley, exposing the strata of a perpendicular bank some forty feet in height. The top of the bank is the general level of the valley, in which was growing an oak tree five feet in diameter. The first eighteen inches in depth is the usual dark surface soil; the next eight feet is yellow clay, apparently of the Loess formation common in Kansas. This clay rests upon a former surface soil two feet thick, of rich black loam; the line between the two is sharply defined. The black soil merges into clay below, which extends down to gravel resting upon the bed-rock of the river. On the surface of the black soil and under the eight feet of clay I found the remains of a camp, containing broken pottery, charcoal, ashes, burnt bones, and stones such as would be used in a camp fireplace. The bones resembled the leg-bones of deer. The black stratum of soil was undoubtedly the surface of the valley, rich in vegetable and animal life, at the time the aboriginal, antediluvian people feasted around their camp-fire, and broke their soup-bowl.

We are evidently not the first settlers of Kansas.

DIFFERENTIALS OF THE SECOND AND HIGHER ORDERS.

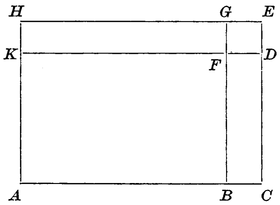
BY E. MILLER, LAWRENCE.

This memoir is written as an answer to one of many questions that have been addressed to the Department of Mathematics of the University. Such inquiries are of almost weekly occurrence. They cover ground that extends all the way from Problems in Percentage to "Curves of Pursuit." They embrace the "settlement of estates;" disputes arising from the "foreclosure of mortgages;" the winding-up of "joint-stock companies;" the contents of cisterns; the "horse power" of mill-dams; the properties of various kinds of curves; the Theory of Probabilities, and the elimination of differentials. More than once has the chair of mathematics been called upon to elaborate the principles, the notation, and the application of differentials, and to show why all differential expressions, of second or higher orders, when compared with those of the first order or degree, vanish completely from the work in hand, without affecting the result. To have answered this question in detail, by unfolding the demonstration step by step, as generally given by either of the methods of the Calculus, would have required more time and leisure than I had at my command. The proposer of the question was evidently laboring under the impression that to throw away the quantity $dxdy$ from such an expression as $xdy+ydx+dxdy$, was an absurd thing to do; believing that if dx and dy each had a value, however small, then $dxdy$ would also have a value, although indefinitely smaller than the former. Now, whether we use the Infinitesimal Method, in which "a quantity is conceived under such a form, or law, as to be necessarily less than any assignable quantity," and according to which infinitesimals of the *second*, *third*, and *higher* orders, may be dropped as not affecting the result; or, the "Method of Limits," as enunciated by one of the discoverers of the Calculus, that "quantities, and ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer the one to the other than by any difference, become ultimately equal," *we know* that dropping the higher orders will give beyond a peradventure the exact result.

But without discussing the merits or the principles of either of these methods, it is proposed in this memoir to furnish a method that is rigorously exact, and does not depend upon any suppositions or theories, fictitious or otherwise. Sir Isaac Newton based his Calculus, or system of fluxions, as he called it, upon the doctrine of limits; but by reference to his Principia, Book II, Section II, Lemma II, it may be seen that there was in his mind a better and a rigorous method by which to obtain the desired results.

In order to illustrate the superiority of the Newtonian method over all others, take for example the proposition that the differential of the product of two variables is the differential of the first into the second, plus the differential of the second into the first.

First, by the infinitesimal method:

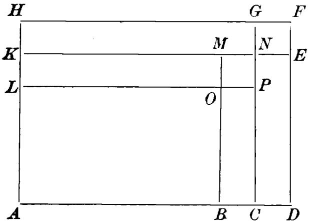


Let AB $=x$, and BF $=y$. Let — BC $=dx$, and FG or DE $=dy$. Then the area of ABFK will be represented by xy. Now, we shall represent the original value of ABFK by the equation $u=xy$.

By taking on increments the consecutive value of the function becomes $u+du=xy+ydx+xdy+dxdy$. Subtracting the first equation from the second, the result becomes $du=ydx+xdy+dxdy$. It is easily seen that ydx and xdy are infinitesimals of the first order, but $dxdy$, being the product of two infinitesimals, each of the first order, constitutes an infinitesimal of the second order, and therefore must be dropped. But $dxdy$ is the area of FDEG, and however small dx and dy may be, their product does represent some value. The method is very pretty, and reaches conclusions with less labor than by the doctrine of limits. Let us now examine Newton's Lemma II. It is as follows: "The moment of any genitum is equal to the moments of each of the generating sides drawn into the indices of the powers of those sides, and into their coëfficients continually." Newton then goes on to explain his Lemma thus: "I call any quantity a genitum which is not made by addition or subtraction of divers parts, but is generated or produced in arithmetic by the multiplication, division, or extraction of the root of any terms whatsoever; in geometry, by the invention of contents and sides, or of the extremes and means of proportionals. Quantities of this kind are products, quotients, roots, rectangles, squares, cubes, square and cube sides, and the like. These quantities I here consider as variable and indetermined, and increasing or decreasing, as it were, by a perpetual motion or flux, and I understand their momentaneous increments or decrements by the name of moments; so that the increments may be esteemed as added or affirmative moments, and the decrements as subducted or negative ones. But take care not to look upon finite particles as such. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the just nascent principles of finite magnitudes. Nor do we in this Lemma regard the magnitude of the moments, but their first proportion, as nascent. It will be the same thing if, instead of moments, we use either the velocities of the increments and decrements (which may also be called the motions, mutations, and fluxions of quantities), or any finite quantities proportional to those velocities. The coëfficient of any generating side is the quantity which arises by applying the genitum to that side." Newton then says that "the sense of the Lemma is, that if the moments of any quantities, A, B, C, etc., increasing or decreasing by a perpetual flux, or the velocities of the mutations which are proportional to them, be called a, b, c, etc., the moment or mutation of the generated rectangle AB will be $aB+bA$; the moment

of the generated content ABC will be $aBC + bAC + cAB$; and the moments of the generated powers A^2, A^3, A^4, $A^{\frac{1}{2}}$, $A^{\frac{3}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{2}{3}}$, A^{-1}, A^{-2}, $A^{-\frac{1}{2}}$, will be $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{3}{2}aA^{\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $\frac{2}{3}aA^{-\frac{1}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, $-\frac{1}{2}aA^{-\frac{3}{2}}$ respectively; and in general, that the moment of any power $A^{\frac{n}{m}}$, will be $\frac{n}{m}aA^{\frac{n-m}{m}}$. Also, that the moment of the generated quantity A^2B will be $2aAB + bA^2$; the moment of the generated quantity $A^3B^4C^2$ will be $3aA^2B^4C^2 + 4bA^3B^3C^2 + 2cA^3B^4C$; and so on."

We shall now illustrate Newton's method by the notation in common use:



Let it be required to find the increment of the rectangle ACNK. Let AC $= x$; CN $= y$; BC $= -\frac{1}{2}dx$; CD $= +\frac{1}{2}dx$; MO $=$ NP $= -\frac{1}{2}dy$; NG $=$ EF $= +\frac{1}{2}dy$. Now, let xy represent the value of the rectangle when it has reached ACNK. First, giving to x and y the increments $+\frac{1}{2}dx$ and $+\frac{1}{2}dy$, respectively, and *then* the decrements $-\frac{1}{2}dx$ and $-\frac{1}{2}dy$, we shall have:

(*a*). $(x + \frac{1}{2}dx)\ (y + \frac{1}{2}dy) = xy + \frac{1}{2}ydx + \frac{1}{2}xdy + dx.dy.$

(*b*). $(x - \frac{1}{2}dx)\ (y - \frac{1}{2}dy) = xy - \frac{1}{2}ydx - \frac{1}{2}xdy + dx.dy.$

Subtract (*b*) from (*a*), and there will remain the absolute increase of the rectangle, *i. e.*, $ydx + xdy$. In a similar manner, it can be easily shown that the increment of the volume xyz is $xy.dz + xz.dy + yz.dx$. Newton's method, therefore, is vastly superior to the others, in that it disposes of $dx.dy$, $dx.dz$, $dy\,dz$, $dx.dy.dz$, dx^2, dy^2, etc., by a mathematical demonstration that is rigorously exact. That is to say, when those higher differentials are used in connection with differentials of the first order.

AN ELECTRICAL HYGROMETER.

[*Abstract.*]

BY LUCIEN I. BLAKE, LAWRENCE.

An hygroscopic substance, as chloride of zinc, is made the electrolyte in a galvanic cell. Variations in the amount of moisture in the atmosphere will alter the amount of this electrolyte and consequently the internal resistance of this cell. The cell itself is conveniently made by a strip of zinc and one of copper bridged by a piece of filter paper soaked in $ZuCl_2$. By connecting the poles through a sensitive galvanometer the deflections may be used as an indication of the amount of moisture present in the air. After five hours the polarization is about 20 per cent., hence it is practically *nil* during the time of an observation. Comparisons have been made for about a week with a Regnault's hygrometer. While the deflections followed the Regnault's with regularity, as yet the law connecting the amount of moisture and the deflections of the galvanometer has not been fully established. It is believed the instrument may be employed for a simple, inexpensive, and fairly accurate method of hygrometric observations.

FACIAL EXPRESSION AND ITS PSYCHOLOGY.

BY A. H. THOMPSON, TOPEKA, KANSAS.

The human face is said to be the mirror of the soul, because it reflects not only the static intelligence and refinement of the mind, but also betrays its transient emotions and passing impulses. The face is the servant of the emotions. It mir-